Abstracts

Speaker: Hideto Asashiba, Shizuoka University
Title: *Interval replacements of persistence modules*
Abstract: We define two notions. The first one is a **compression system** ξ for a finite poset P, which assigns to each interval subposet I an order-preserving map $\xi_I \colon I^\xi \to P$ satisfying some conditions, where I^ξ is a connected finite poset.
An example is given by the **total** compression system that assigns to each I the inclusion of I into P. The second one is an **I-rank** of a persistence module M under ξ, the family of which is called the **interval rank invariant** of M under ξ.

A compression system ξ makes it possible to define the **interval replacement** (also called the interval-decomposable approximation) not only for 2D persistence modules but also for any persistence modules over any finite poset.

We will show that the forming of the interval replacement preserves the interval rank invariant, which is a stronger property than the preservation of the usual rank invariant. Moreover, to know explicitly what is preserved, we will give a formula of the I-rank of M under ξ in terms of the structure linear maps of M for any compression system ξ. This makes it possible to define an essential cover property for a compression system, and by using this notion we give a sufficient condition for the I-rank of M under ξ to coincide with that under the total compression system, the value of which is equal to the generalized rank invariant introduced by Kim–Mémoli.
This is joint work with Etienne Gauthier and Enhao Liu.

Speaker: Saugata Basu, Purdue
Title: *New quantitative results in semi-algebraic geometry/topology with applications*
Abstract: I will discuss two recent quantitative results that are potentially of interest in applied topology.

The first result is an optimal upper bound on a higher degree cohomological version of Vapnik-Chervonenkis (VC) density. The classical VC density and known upper bounds are recovered as the degree 0 case. I will discuss potential applications of this theorem for topological sampling via a higher degree version of ε-net theory. (*This is joint work with D. Patel.*)

In the second part, I will introduce the notion of speed of a family of finite poset modules originating from some fixed multi-parameter semi-algebraic filtration. The speed measures the growth of the number of isomorphism classes of the poset modules in the family as the size of the poset increases. We prove an upper bound on the speed that is asymptotically of the same shape as the speed of semi-algebraically defined graph classes (such classes of graphs have been studied extensively in graph theory). (*Joint work with A. Banerjee.*)

Speakers: Nadya Belova, Max Goldberg, Andrew Xie, Rutgers
Title: *Discrimination of dynamic data via curvature sets*
Abstract: Techniques from topological data analysis (TDA) have proven highly effective in studying time-dependent data arising in dynamic systems, such as animal swarming behavior and spatiotemporal patterns in neuroscience. While early algorithms, such as those introduced by Munch, leveraged efficient updates to persistence diagrams for dynamic data, they struggled to distinguish behaviors that are isometric at each fixed time but differ qualitatively. This limitation was addressed by Kim and Mémoli, who introduced a spatiotemporal persistence framework for dynamic metric spaces, resulting in multiparameter persistence modules. However, these modules pose computational challenges.

To address this, we build on insights from Gómez and Mémoli, who observed that the homology of Vietoris–Rips complexes over size-2k+2 point subsets of a metric space—termed curvature sets—is both tractable and informative. We extend this idea to dynamic settings by introducing dynamic curvature-set persistent homology, applying the spatiotemporal framework of Kim and Mémoli to curvature sets. The resulting modules are interval-decomposable and admit efficient algorithms for computing interleaving distances. Our construction is stable with respect to the dynamic Gromov–Hausdorff distance introduced by the same authors, and we implement both the erosion distance d_E (due to Puuska) and a new Hausdorff-inspired distance d_2 with cubic and quadratic runtime, respectively. This enables a robust computational pipeline for distinguishing dynamic data, as demonstrated in experiments with the boids model, where we successfully detect parameter changes.

Speaker: Nate Clause, OSU
Title: *The Generalized Rank Invariant: Möbius invertibility, Discriminating Power, and Connections to Other Invariants*
Abstract: The recently introduced notion of generalized rank invariant (GRI) naturally extends the rank invariant by enlarging the domain to the collection of all connected sub-posets of an indexing poset. Thus, the GRI acquires more discriminating power than the rank invariant, but at the expense of computational cost.

In this talk, we will study the tension that exists between computational efficiency and the discriminating power of the GRI as one varies the size of the restricted domain set of the GRI. To do so, we introduce the notion of Möbius invertibility of the GRI, which allows us to utilize the theory of Möbius inversion to characterize the discriminating power of a restricted GRI based on its domain set. Importantly, the notion of Möbius invertibility allows us to utilize key results from the classical theory of Möbius inversion, even when the indexing poset for a persistence module is not necessarily locally finite. We will conclude by comparing the GRI and Möbius invertibility of the GRI to other existing invariants and notions of tameness for multiparameter persistence from the literature.

Speaker: Nir Gadish, UPenn
Title: *Hömotopical foundations of Möbius inversion*
Abstract: Möbius inversion for posets was discovered as a combinatorial construction, but in 1936 Hall observed that the values of the Möbius function are Euler characteristics of intervals in the poset, suggesting that homotopy theory is at play. Möbius homology clarifies and categorifies much of the combinatorial theory, but it can be lifted further!

In this talk we will discuss a functorial "space-level" realization of Möbius inversion for diagrams taking values in any (infinity-) category with some basic properties. The role of the Möbius function will be played by homotopy types whose homology is the Möbius homology, and so their Euler characteristic is the classical Möbius function.

Notable manifestations of this phenomenon include handle decompositions of manifolds, Koszul resolutions of algebras, and filtrations of configuration spaces.

Speaker: Rob Ghrist, UPenn
Title: *Network Sheaves & Torsors*
Abstract: Network sheaves are the simplest type of cellular sheaf—almost so simple as to be overlooked. Nevertheless, a combination of cohomology, Hodge theory, and obstruction theory (in the form of "network torsors") is enough to model a surprising array of useful constructs. This talk will survey a number of novel and applicable instances, with examples.

This talk will be accessible to students who are new to applied algebraic topology, and also interesting to experts.

Speaker: Woojin Kim, KAIST
Title: *Complexity and Estimation of the Generalized Rank Invariant and Beyond*
Abstract: The Generalized Rank Invariant is a natural invariant of multi-parameter persistence modules and represents one possible extension of the persistence diagram to the multi-parameter setting. In this talk, we will explore its structural and computational complexity, estimation methods, and, time permitting, its comparison with other “barcoding invariants.”
The talk is based on three joint works: (1) with Donghan Kim and Wonjun Lee, (2) with Mathieu Carrièr and Seunghyunk Kim, and (3) with Emerson G. Escolar.

Speaker: Alex McCleary, Montana State
Title: *Filtered Moore Spaces*
Abstract: The study of persistence modules over posets has proven to be quite challenging. As a result, there has been significant research into a special case of multiparameter persistence: one-critical filtrations. We show that much of the study of persistence modules over posets reduces to the study of one-critical filtrations with no loss of generality. We accomplish this by providing a construction of filtered Moore spaces for persistence modules that is compatible with interleavings.

Concretely, we show that if P is a poset with a minimum element and M is a persistence module over P, then for any $n > 0$, there exists a CW complex X and a function $f: X \to P$ such that $H_n \circ f{\downarrow} \cong M$ where $f{\downarrow}$ is the sublevel set filtration of f. Moreover, if M_0 and M_1 are ε-interleaved persistence modules over P, then for any $n > 0$, there exists a CW complex X and functions $f_0, f_1: X \to P$ such that $H_n \circ f_0{\downarrow} \cong M_0$, $H_n \circ f_1{\downarrow} \cong M_1$, and $||f_0 - f_1||_\infty = \varepsilon$.

Speaker: **Amit Patel, Colorado State.**

Talk I (Monday) – Title: *From Classical Persistence to Möbius Inversion: An Introduction*
Abstract: In this two-part introduction, I will motivate and develop the theory of persistent homology from a categorical and combinatorial perspective. We will focus on classical 1-parameter persistence, where persistence modules are typically valued in vector spaces, and reinterpret its familiar constructions—such as barcodes and the bottleneck distance—in the more general setting of persistence modules valued in an arbitrary abelian category. Along the way, I will present examples and pose open problems.

A central theme will be Möbius inversion, which originates in poset combinatorics but has emerged as a useful organizing principle in persistence theory. I will briefly outline how Möbius-theoretic methods extend to the multiparameter setting, setting the stage for several talks later in the workshop.

These talks are intended to provide both background and orientation for the week. I will highlight conceptual connections to upcoming presentations, including the generalized rank invariant, interval-decomposable approximations, Möbius homology, and persistent Laplacians.

Talk II (Tuesday) -- Title: *From Möbius Inversion to Möbius Homology*
Abstract: This talk develops the theory of Möbius homology, a homological refinement of classical Möbius inversion on posets. We begin with a motivating idea: numerical Möbius inversion often arises from richer algebraic or topological structures, suggesting the existence of a homology theory whose Euler characteristic recovers the classical Möbius function.

I will construct the Möbius chain complex associated to a functor from a finite poset to an abelian category and define its homology. A central result is Rota's Galois connection theorem, which I will present in this homological framework.

To conclude, I will briefly sketch how Möbius homology offers structural insights into persistence theory, particularly in the multiparameter setting, and how it connects to ongoing efforts to understand generalized rank invariants and stability.

Speaker: Tatum Rask, CSU

Title: *Persistent Laplacians & Möbius Homology*

Abstract: In this talk, we will introduce a new notion of a persistent Laplacian: the Laplacian obtained from the Möbius chain complex (as defined by Patel and Skraba). Given a 1-parameter filtration of a simplicial complex, the birth-death module assigns to each interval a space of cycles born at or before the start of the interval that die by the end of the interval. By constructing the Möbius chain complex for the birth-death module at a specified interval, we can compute a combinatorial (cosheaf) Laplacian.

The kernel of this Laplacian, much like other notions of persistent Laplacians, provides a space of cycle representatives for that interval in the persistence diagram. Further, we present preliminary results suggesting that the spectrum of this Laplacian reflects the structure of our persistence diagram.

Speaker: Zhengchao Wan, Missouri

Title: *Grassmannian Persistence Diagrams*

Abstract: In this talk I will introduce Grassmannian Persistence Diagrams (GPDs), a new class of vector space–valued invariants that generalize classical persistence diagrams.

We show that GPDs can be equivalently defined by applying suitable variants of the Möbius inversion to either birth–death spaces or persistent Laplacian kernels. Moreover, we construct an explicit projection that yields an isomorphism between GPDs and the harmonic barcodes introduced by Basu and Cox.

We point out that the GPDs are able to retain more information about the underlying simplicial filtrations than classical persistence diagrams. For example, we show that degree-0 Grassmannian persistence diagrams are equivalent to treegrams, a generalization of dendrograms. As a consequence, we show that finite ultrametric spaces can be fully reconstructed from the degree-0 GPDs associated with their Vietoris–Rips filtrations—a task that is not possible using classical persistence diagrams alone.

This is joint work with Aziz Burak Gülen and Facundo Mémoli.

Speaker: Qingsong Wang, UCSD
Title: *Tight Spans as a Lens on Vietoris–Rips Topology*
Abstract: The tight span of a metric space X, denoted E(X), represents the smallest hyperconvex metric space containing X and serves as a powerful tool for understanding the behavior of Vietoris–Rips complexes.

We show how classic results like Hausmann's theorem for general Riemannian manifolds can be recovered from this hyperconvex perspective for small scale. On the other hand, for large scale cases in the case of dense subsets of Euclidean space with ℓ_1 metric, particularly integer lattices, we demonstrate how hyperconvex embeddings provide a clean approach to analyzing contractibility properties of Vietoris–Rips complexes.

Speaker: Ling Zhou, Duke
Title: *Beyond Persistent Homology: Persistent Cup Products and More Discriminative Persistent Invariants*
Abstract: Persistent homology is a foundational tool in topological and geometric data analysis, capturing multiscale topological features of data. However, its discriminative power is limited in certain contexts. In this talk, I present a broader framework of persistent invariants that extends the power of persistent homology.

A central focus is on persistent cup products, which capture higher-order interactions missed by homology alone. I will discuss both the theoretical foundations, including stability results, and practical applications, such as detecting toroidal structures in neural data. I will also outline connections to other enriched invariants, including those derived from the LS-category, topological complexity, and Steenrod operations, pointing toward a unified approach to richer topological signatures in data.

In certain cases, we show that these enhanced invariants yield stronger lower bounds on the Gromov–Hausdorff distance between metric spaces than standard persistent homology.